Determination of Supersonic Panel Flutter of Cylindrical Shells With In-Plane Stresses

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In a recent paper, Holt and Strack[1] present a formula based on the piston theory for determining the minimum thickness necessary to prevent panel flutter of cylindrical shells for $M > 2$. Hedgepeth[2] has presented a method of utilizing the piston theory for rectangular, simply supported plates. The piston theory has been utilized by a number of authors.[3, 4]

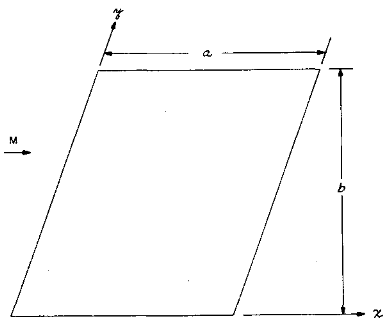

Fig. 1. Flat plate with aspect ratio b/l subjected to velocity flow.

Consider a flat plate (Fig. 1), with the flow as shown. For a cylindrical full panel (Fig. 2), $b = 2\pi r$. From Hedgepeth's paper,[2] the critical dynamic pressure λ is given as

$$\lambda_{cr} = 2qa^3/\beta D \tag{1}$$

where

q = dynamic pressure = $(1/2)\rho u^2$
β = $\sqrt{M^2 - 1}$
D = plate stiffness = $Eh^3/12(1 - \mu^2)$
μ = Poisson ratio
M = Mach number = μ/c
c = speed of sound

Substituting into Eq. (1) and simplifying,

$$\lambda_{cr} = \rho c^2 M^2 a^3 / D\sqrt{M^2 - 1} \tag{2}$$

Since

$$c^2 = \gamma P/\rho$$

then

$$\lambda_{cr} = M^2 a^3 \gamma P / D(M^2 - 1)^{1/2} \tag{3}$$

If one considers the cylinder to be stretched out to a plate and if $b \gg l$, this may be considered as a plate with an infinite aspect ratio. From Hedgepeth,[2] $\lambda_{cr} = 341$. If no midplane stresses are considered and substituting for λ_{cr} and D in Eq. (3), the following results:

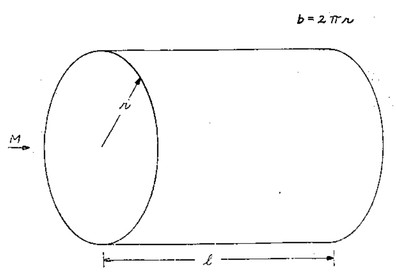

Fig. 2. Cylindrical shell subjected to velocity flow.

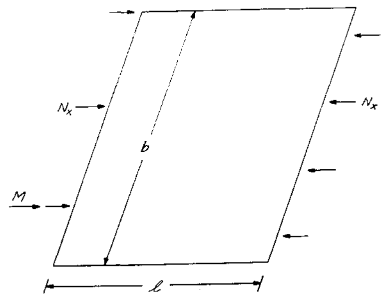

Fig. 3. Flat plate subjected to velocity flow with in-plane stresses.

$$\frac{h}{l} = 0.3276\,(1 - \mu^2)^{1/3}\left(\frac{\gamma P}{E}\right)^{1/3}\frac{M^{2/3}}{(M^2 - 1)^{1/6}} \tag{4}$$

This compares favorably with Holt and Strack[1] for a cylinder with simply supported ends—viz.,

$$\frac{h}{l} = 0.3218\,(1 - \mu^2)^{1/3}\left(\frac{\gamma P}{E}\right)^{1/3}\frac{M^{2/3}}{(M^2 - 1)^{1/6}} \tag{5}$$

Eq. (3) can be further extended to multilayer cylinders. Should in-plane stresses be considered (Fig. 3), Hedgepeth[2] derives certain parameters namely $\bar{A}$ and R_x where

$$\bar{A} = \bar{R}_x - 2(l/b)^2 \tag{6}$$

$$\bar{R}_x = R_x/\pi^2 = N_x l^2/D\pi^2 \tag{7}$$

Substituting Eq. (7) into Eq. (6)

$$\bar{A} = \frac{N_x l^2}{D\pi^2} - 2(l/b)^2 \cong \frac{N_x l^2}{D\pi^2} \tag{8}$$

where $N_x = F/a$ (force/unit span and the compressive force is considered positive).

Utilizing the same previous assumptions then Eq. (4) is modified to

$$\frac{h}{l} = K(1 - \mu^2)^{1/3}\left(\frac{\gamma P}{E}\right)^{1/3}\frac{M^{2/3}}{(M^2 - 1)^{1/6}} \tag{9}$$

where K is tabulated in Table 1.

Table 1.

A	K
−6	0.2405
−4	0.2601
−2	0.2873
0	0.3276
2	0.3977
4	0.5913

In determining the minimum thickness necessary for in-plane stresses, calculate the effective thickness by Eq. (4). If the in-plane stresses are low, this thickness may be sufficient. If F or N_x is large, determine A from Eq. (8) and determine K from Table 1. Determine h from Eq. (9).

References

[1] Holt, M., and Strack, S. L., *Supersonic Panel Flutter of a Cylindrica Shell of Finite Length*, Journal of the Aerospace Sciences, Vol. 28, No. 3, March 1961.

[2] Hedgepeth, J. M., Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds, Journal of the Aeronautical Sciences, Vol. 24, No. 8, Aug. 1957.

[3] Chawla, J., *Aeroelastic Instablity at High Mach Numbers*, Journal of Aerospace Sciences, Vol. 25, pp. 246, April 1958.

[4] Morgan, H. G., Runyan, H. L., and Huckel, V., *Theoretical Consideration of Flutter at High Mach Numbers*, Journal of Aeronautical Sciences, Vol. 25, No. 6, June 1958.